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Extended Lévêque solution for heat transfer to non-Newtonian fluids in pipes and flat ducts

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Abstract—The problem of predicting the rate of heat transfer through the wall of a duct in which a non-Newtonian fluid is flowing in laminar regime is analyzed in the present contribution. The effect of heat generation by viscous dissipation is also considered.

In this work an alternative route, to deduce an extension of the Lévêque solution, is presented up to terms including the effect of the Brinkman number. The results so obtained are compared with previous results showing that our final expression, valid for pipes as well as flat ducts, produces extremely good results within the range of validity of the variables which is specified.

INTRODUCTION

As POINTED out recently by Richardson [1], there are a number of practical situations where the rate of heat transfer from the walls to a fluid flowing in a fully developed laminar regime must be predicted. The analysis of such systems, with the usual assumptions, normally leads to the well known generalized Graetz–Nusselt equation where the rheological fluid behavior and the rate of viscous dissipation are also considered.

Although this problem has been the main subject of quite a number of investigations, no simple solution has yet been found. The normal procedure of variable separation leads to a reasonable number of eigenvalues for each particular set of rheological parameters and Brinkman number. Moreover, the resulting solution series does not converge very rapidly when the duct length is very short since a considerable number of eigenvalues would be required. The problem is overcome by searching for a precise analytical solution valid for the first step of the developing boundary layer. This was the main scope of many analytical investigations in which the famous Lévêque [2] solution was extended to take into account, through a perturbation procedure, further terms up to those in which the effect of the Brinkman number is taken into account.

When the Lévêque [2] solution was considered as zeroth order, Newman [3] was able to find the analytical solution up to second-order terms for the case of a Newtonian fluid with a negligible viscous dissipation rate. Shih and Tsou [4] extended the solution up to fourth-order terms considering the case of a power-law non-Newtonian fluid as well as the effect of Brinkman number. However, their procedure implies the numerical solution of a set of ordinary linear differential equations which must be solved for each particular set of parameters. Richardson [1] was able to extend Newman's [3] procedure up to the third-order terms for the case of a power-law non-Newtonian fluid under the effect of viscous dissipation. Two asymptotic expressions for the mean Nusselt number were presented: one for circular and the other for rectangular ducts. This procedure involved the use of hypergeometric (Kummer) functions and their properties.

In this short contribution it will be shown that with a very simple procedure, developed very recently by Gottifredi *et al.* [5], it is possible to obtain similar expressions as those derived by Richardson [1] for the mean Nusselt number without getting involved with complex hypergeometric functions. A unique expression will be derived which is valid for both plane and cylindrical geometries. Moreover, since the expressions are first obtained in a Laplacian transformed field they can be used in the matching procedure previously described [5] to obtain a unique approximate expression valid in the whole range of duct lengths.

ANALYSIS

Assuming laminar flow, constant physical properties of the fluid and a power-law model for describing the rheological behavior of the fluid the dimensionless energy balance can be written as:

$$(1-Y^{N+1})\frac{\partial T}{\partial Z} = \frac{1}{Y^M}\frac{\partial}{\partial Y}\left(Y^M\frac{\partial T}{\partial Y}\right) + Br\,Y^{N+1} \quad (1)$$

where:

$$T = \frac{t-t_0}{t_s-t_0}; \quad Y = \frac{r}{R} \quad (2a,b)$$

$$Z = n(M+1)(x/R)k_T/(\rho C_p\langle V\rangle R) \quad (3)$$

$$n = (M+1)^{-1} - (M+N+2)^{-1} \quad (4)$$

$$Br = \frac{b}{k_T(t_s-t_0)}[\langle V\rangle(M+N+2)]^{\frac{N+1}{N}} \Big/ R^{\frac{1-N}{N}} \quad (5)$$

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NOMENCLATURE

A	coefficient in equation (22)
$Ai(\ )$	Airy function
B	coefficient in equation (22)
b	shear rate viscosity
$Bi(\ )$	Biot function
Br	Brinkman number
C	coefficient in equation (22)
C_p	specific heat capacity
D	coefficient in equation (22)
d_0	coefficient in equation (25)
d_3	coefficient in equation (25)
F_0	first term in equation (11)
F_1	first perturbation term in equation (11)
F_2	second perturbation term in equation (11)
g	parameter defined by equation (30)
$Gi(\ )$	function defined by equation (28)
Gz	Graetz number
h	parameter defined by equation (32)
h_T	average heat transfer coefficient
k_T	thermal conductivity
L	duct length
M	constant to denote flat (0) or cylindrical geometry (1)
N	constant index in power-law model
Nu	Nusselt number
n	parameter defined by equation (4)
p	parameter defined by equation (31)
Q	volumetric flow rate
R	tube radius or half thickness of a flat duct
r	normal coordinate measured from the axial axis of the duct
S	Laplace transform variable
T	dimensionless temperature
T_m	dimensionless mixing cup temperature
t	dimensional temperature
t_0	inlet fluid temperature
t_s	wall temperature
U_1	function given by equation (20a)
V_1	function given by equation (20b)
$\langle V \rangle$	average velocity in the duct
W	rectangular channel width
X	transformed variable of Y when $S \to \infty$
x	axial flow coordinate
Y	dimensionless radial (pipe) or cross-channel (flat duct) coordinate
y	variable in equation (28)
Z	dimensionless axial coordinate
Z_L	value of Z at $x = L$
z	constant in equation (28).

Greek symbols

$\Gamma(\ )$	gamma function	
ζ	transformed axial distance	given by Shih and Tsou [4]
θ	dimensionless temperature	given by Shih and Tsou [4]
θ_i	coefficients of the power series of ($i = 0, 1, 2$)	given by Shih and Tsou [4]
ξ	transformed radial distance	given by Shih and Tsou [4]
ρ	density	
ϕ	Laplace transform of T	
Ψ_0, Ψ_1	components of F_2.	

Superscripts

$'$	first derivative
$''$	second-order derivative.

t being the dimensional temperature, x the axial flow coordinate, k_T the thermal conductivity of the fluid, $\langle V \rangle$ the average velocity in the duct, R the radius of the pipe or half thickness of a flat duct, C_p the specific heat capacity of the fluid, ρ the fluid density, b the shear rate viscosity, N the viscosity shear rate exponent and M to denote plane ($M = 0$) or cylindrical ($M = 1$) geometry of the duct. It should be noted that Br can be positive or negative according to the value of $(t_s - t_0)$. Thus if the flow is being cooled $Br < 0$ while the opposite situation is met when the fluid is heated.

The problem will be solved subject to the following boundary conditions:

$$T = 0 \qquad Z = 0 \qquad 1 > Y \geqslant 0 \tag{6a}$$

$$\partial T/\partial Y = 0 \qquad Z \geqslant 0 \qquad Y = 0 \tag{6b}$$

$$T = 1 \qquad Z \geqslant 0 \qquad Y = 1 \tag{6c}$$

although, since the problem is linear, it could very well be solved with other boundary conditions if we wanted to do so.

Following the procedure of Gottifredi *et al.* [5], equation (1) can be most conveniently transformed by introducing:

$$\phi = S \int_0^{\infty} T \exp(-SZ)\,dZ \tag{7}$$

which reduces equation (1) to the following ordinary differential equation:

$$\frac{d}{dY}\left(Y^M \frac{d\phi}{dY}\right) = Y^M(1 - Y^{N+1})S\phi - Br\,Y^{M+N+1}. \tag{8}$$

We are mainly interested in searching for an asymptotic solution to equation (8) when $Z \ll 1$ ($S \to \infty$) up to those terms where the influence of Br is noticeable. The main difference between our procedure and that used by Richardson [1] is that we perform the transformation given by equation (7) which leads to much simpler ordinary differential equations than the basic ones deduced by Richardson [1].

By introducing a new independent variable (see Gottifredi *et al.* [5]):

$$X = S^{1/3}(1-Y) \tag{9}$$

and placing it into equation (8) yields:

$$S^{2/3}\frac{d^2\phi}{dX^2} - MS^{1/3}(1+XS^{-1/3}+X^2S^{-2/3}+\cdots)\frac{d\phi}{dX}$$
$$= [(N+1)XS^{-1/3} - \tfrac{1}{2}N(N+1)X^2S^{-2/3} + \tfrac{1}{6}(N+1)N(N-1)X^3S^{-1} - \cdots]S\phi - Br[1-(N+1)XS^{-1/3}+\cdots] \tag{10}$$

which suggests the following series solution for ϕ:

$$\phi = F_0(X) + S^{-1/3}F_1(X) + S^{-2/3}F_2(X). \tag{11}$$

By substituting equation (11) into equation (10) and after collecting terms of equal power of S the following system of coupled ordinary differential equations is generated:

$$F_0'' = (N+1)XF_0 \tag{12}$$

$$F_1'' = (N+1)XF_1 + MF_0' - \tfrac{1}{2}N(N+1)X^2F_0 \tag{13}$$

$$F_2'' = (N+1)XF_2 + MF_1' - \tfrac{1}{2}N(N+1)X^2F_1 + \tfrac{1}{6}(N+1)N(N-1)X^3F_0 + MXF_0' - Br \tag{14}$$

subject to:

$$F_0(0) = 1;\quad F_1(0) = F_2(0) = F_0(\infty) = F_1(\infty) = F_2(\infty) = 0. \tag{15}$$

The solution to equation (12) subject to the corresponding boundary conditions, defined by equation (15) is well known and can most conveniently be written in terms of the Airy function (see Antosiewicz [6])

$$F_0 = Ai((N+1)^{1/3}X)/Ai(0) \tag{16}$$

while equation (13) only needs a particular solution since the solution to the homogeneous equation is trivial.

Thus, the following particular solution can be tried:

$$F_1 = U_1(X)\cdot F_0 + V_1(X)\cdot F_0' \tag{17}$$

and it can be shown that U_1 and V_1 must fulfil:

$$U_1'' + (N+1)V_1 + 2(N+1)XV_1' = -\tfrac{1}{2}N(N+1)X^2 \tag{18a}$$

$$2U_1' + V_1'' = M \tag{18b}$$

so that:

$$-\tfrac{1}{2}V_1''' + (N+1)V_1 + 2(N+1)XV_1' = -\tfrac{1}{2}N(N+1)X^2. \tag{19}$$

Thus:

$$V_1 = -0.1NX^2 \quad \text{and} \quad U_1 = (0.5M+0.1N)X \tag{20a,b}$$

and the appropriate boundary conditions for F_1 are then fulfilled.

Regarding equation (14), it should first be noticed that F_1 and F_1' can be replaced as functions of F_0 and F_0' and that the non-homogeneous part due to the effect of Br can be separated by assuming:

$$F_2 = \Psi_0(X) + Br\Psi_1(X). \tag{21}$$

By substituting F_1 and F_1' (in terms of F_0 and F_0') and F_2 given by equation (21) into equation (14) it yields:

$$\Psi_0'' = (N+1)X\Psi_0 + (A+BX^3)F_0 + (CX-DX^4)F_0' \tag{22}$$

$$\Psi_1'' = (N+1)X\Psi_1 - 1 \tag{23}$$

where:

$$A = (0.5M+0.1N)M \tag{24a}$$

$$B = \tfrac{1}{6}(N+1)N(N-1) - 0.1N(N+1)M - \tfrac{1}{2}(0.5M+0.1N)N(N+1) \tag{24b}$$

$$C = (1+0.5M-0.1N)M \tag{24c}$$

$$D = -0.05N^2(N+1). \tag{24d}$$

Equation (22) can be solved by following exactly the same procedure as that used above for equation (13), resulting in:

$$\Psi_0 = \left[\left(\frac{C}{2}-3d_3\right)\frac{X^2}{2} - 0.1DX^5 - d_0(N+1)^{1/3}Ai'(0)/Ai(0)\right]F_0 + [d_0+d_3X^3]F_0' \tag{25}$$

with:

$$d_0 = (A-0.5C+3d_3)/(N+1) \tag{26a}$$

$$d_3 = (B+2D)/(7(N+1)) \tag{26b}$$

while $\Psi_0(0) = \Psi_0(\infty) = 0$. The solution of equation (23) is given by Antosiewicz [6]:

$$\Psi_1 = \frac{\pi}{(N+1)^{2/3}}[Gi((N+1)^{1/3}X) - Gi(0)Ai((N+1)^{1/3}X)/Ai(0)] \tag{27}$$

where Gi is given by:

$$Gi(z) = \tfrac{1}{3}Bi(z) + \int_0^z [Ai(z)Bi(y) - Ai(y)Bi(z)]\,dy. \tag{28}$$

Bi being the Biot function as defined by Antosiewicz [6]. Ψ_1 as given by equation (27) vanishes both when $X \to 0$ and also when $X \to \infty$.

Finally, the calculation of $(d\phi/dY)$ can be performed, yielding:

$$\frac{1}{S}\frac{d\phi}{dY}\bigg|_{Y=1} = gS^{-2/3} - pS^{-1} + hS^{-4/3}. \tag{29}$$

Where:

$$g = -(N+1)^{1/3}Ai'(0)/Ai(0) \tag{30}$$

$$p = 0.5M + 0.1N \tag{31}$$

$$h = d_0(N+1)^{2/3}[Ai'(0)/Ai(0)]^2 + Br\left[\frac{2\pi}{\sqrt{3}}\frac{Ai'(0)}{(N+1)^{1/3}}\right] \tag{32}$$

Table 1. Comparison of values of $\theta_i'(0)$ ($i = 0, 1, 2$) for various N and Br for flow in a pipe

		$\theta_0'(0)$		$\theta_1'(0)$		$\theta_2'(0)$	
N	Br	This work and [1]	([4]	This work and [1]	[4]	This work and [1]	[4]
	0	+1.017448	+1.01745	−0.550000	−0.55000	−0.089413	−0.08941
1/2	1	+1.017448	+1.01745	−0.550000	−0.55000	+0.466926	+0.45
	5	+1.017448	+1.01745	−0.550000	−0.55000	+2.692285	+2.68571
	0	+1.119847	+1.11985	−0.600000	−0.60000	−0.089923	−0.08992
1	1	+1.119847	+1.11985	−0.600000	−0.60000	+0.415545	+0.40
	5	+1.119847	+1.11985	−0.600000	−0.60000	+2.437418	+2.41671
	0	+1.281904	+1.28190	−0.700000	−0.70000	−0.091052	−0.09105
2	1	+1.281904	+1.28190	−0.700000	−0.70000	+0.350515	+0.35
	5	+1.281904	+1.28190	−0.700000	−0.70000	+2.116783	+2.10471

and the local heat flux:

$$\left.\frac{\partial T}{\partial Y}\right|_{Y=1} = \frac{g}{\Gamma(2/3)}Z^{-1/3} - p + \frac{h}{\Gamma(4/3)}Z^{1/3} + \cdots. \quad (33)$$

By defining the mean Nusselt number based on an average heat transfer coefficient (h_T) as:

$$Nu = \frac{2Rh_T}{k_T} = \frac{2\rho C_p\langle V\rangle R^2}{n(M+1)k_T L}\int_0^{Z_L}\left.\frac{\partial T}{\partial Y}\right|_{Y=1} dZ \quad (34)$$

where Z_L denotes the value of Z at $x = L$, and the Graetz number as:

$$Gz = \frac{4\rho C_p Q}{\pi k_T L} \quad \text{or} \quad Gz = \frac{2\rho C_p Q}{(W/R)k_T L} \quad (35a,b)$$

for $M = 1$ or $M = 0$ respectively where W denotes the rectangular channel width, we may now rewrite equation (34) as:

$$Nu = \frac{3g}{\Gamma(2/3)[4n(M+1)]^{1/3}}Gz^{1/3} - 2p + \frac{3h[n(M+1)]^{1/3}}{4^{1/6}\Gamma(4/3)}Gz^{-1/3} \quad (36)$$

which is valid both for rectangular as well as cylindrical channels and can be easily compared with corresponding equations (31) and (55) of Richardson [1].

Our results can be more conveniently compared with previous findings in terms of the local heat flux. Taking into account the definitions of the dimensionless temperature (θ), of the transformed radial distance (ξ) and of the transformed axial distance (ζ) given by Shih and Tsou [4] we can write:

$$\left.\frac{\partial T}{\partial Y}\right|_{Y=1} = \left(\frac{2}{9Z}\right)^{1/3}\left.\frac{\partial \theta}{\partial \xi}\right|_{\xi=0} \quad (37)$$

and considering the power series solution for θ proposed by Shih and Tsou [4] it yields:

$$\left.\frac{\partial \theta}{\partial \xi}\right|_{\xi=0} = \theta_0'(0) + \zeta\theta_1'(0) + \zeta^2\theta_2'(0) + \cdots. \quad (38)$$

By placing equations (33) and (38) into equation (37), and after collecting terms of equal power of Z the following results are generated:

$$\theta_0'(0) = \left(\frac{9}{2}\right)^{1/3}\frac{g}{\Gamma(2/3)} \quad (39a)$$

$$\theta_1'(0) = -p \quad (39b)$$

$$\theta_2'(0) = \left(\frac{2}{9}\right)^{1/3}\frac{h}{\Gamma(4/3)}. \quad (39c)$$

Values of $\theta_i'(0)$ ($i = 0, 1, 2$) are compared in Table 1 with previous findings for flow in a pipe and in Table 2 for flow in a rectangular channel with $N = 1$ and $Br = 0$. As expected they coincide exactly with those of Richardson [1].

Comparing our results with those of Wichterle and coworkers [8, 9] in terms of a mixed mean temperature, from the energy balance (equation (1)) we can write:

$$T_m = \frac{1}{n}\int_0^Z \left.\frac{\partial T}{\partial Y}\right|_{Y=1} dZ + Br\left(\frac{M+1}{N+1}\right)Z \quad (40)$$

By placing equation (33) into equation (40) after integrating yields:

$$T_m = \frac{3g}{2n\Gamma(2/3)}Z^{2/3} + \left[Br\left(\frac{M+1}{N+1}\right) - \frac{p}{n}\right]Z + \frac{3h}{4n\Gamma(4/3)}Z^{4/3}. \quad (41)$$

Wichterle and Wein [8] obtained an approximate expression for the mixed mean temperature valid for a pipe, a flat and for an annular duct. In addition Wichterle *et al.* [9] found an expression for the mixed mean temperature, by using the method of separation

Table 2. Comparison of values of $\theta_i'(0)$ ($i = 0, 1, 2$) with $N = 1$ and $Br = 0$ for flow in a channel

	$\theta_0'(0)$	$\theta_1'(0)$	$\theta_2'(0)$
This work and Richardson [1]	+1.119847	−0.100000	−0.018393
Mercer [7]	+1.12250	−0.10012	−0.01832

Table 3. Comparison of values of T_m with $N = 1$ and $Br = 0$ for flow in a pipe

Z	This work	Wichterle and Wein [8]	Wichterle *et al.* [9]
10^{-6}	0.000405	0.000407	—
10^{-5}	0.001865	0.001887	—
10^{-4}	0.008526	0.008730	—
10^{-3}	0.038253	0.039881	—
10^{-2}	0.163943	0.172133	0.165469
10^{-1}	0.616138	0.583892	0.604736

Table 4. Comparison of values of T_m with $N = 1$ and $Br = 0$ for flow in a flat duct

Z	This work	Wichterle and Wein [8]	Wichterle *et al.* [9]
10^{-6}	0.000152	0.000153	—
10^{-5}	0.000691	0.000708	—
10^{-4}	0.003273	0.003283	—
10^{-3}	0.013758	0.015146	—
10^{-2}	0.069265	0.068388	0.087281
10^{-1}	0.312218	0.280216	0.375945

of variables, valid for long contact times for the three geometries mentioned above. However, [8, 9] did not consider the rate of viscous dissipation.

Tables 3 and 4 compare the results of T_m given by our expression (equation (41)) with those of Wichterle and coworkers [8, 9] for a Newtonian fluid flowing in a pipe and in a flat duct respectively.

CONCLUSIONS

An extended Lévêque solution to estimate the rate of heat transfer through the wall of a duct in which flows a non-Newtonian fluid in a fully developed laminar regime, was deduced. The purpose of this contribution is to show an alternative route to deduce this expression which in a previous reference [1] was through a rather cumbersome procedure. The effect of viscous dissipation is also taken into account, although for short ducts it only affects second-order terms.

Nevertheless, for matching purposes it could very well play an important role in finding a solution at intermediate duct lengths.

As expected, our numerical results coincide exactly with those found by ref. [1] (Table 1). It should be stressed however that we have deduced a unique expression valid for both cylindrical and rectangular channels.

In Table 3, it is shown that the expression can be safely used up to values of Z to the order of 0.1, since our results are very similar to those of Wichterle *et al.* [9] when $0.01 < Z \leqslant 0.1$ for the simplest case of $Br = 0$ and $M = N = 1$. First- and second-order terms are shown to be significant when $Z > 0.001$. Almost the same conclusions can be extracted from the analysis of results presented in Table 4 for a flat duct.

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EXTENSION DE LA SOLUTION DE LEVEQUE POUR LE TRANSFERT THERMIQUE DES FLUIDES NON NEWTONIENS DANS LES TUBES ET LES CONDUITES PLATES

Résumé—On analyse le problème du calcul des flux de chaleur a travers la paroi d'une conduite à l'intérieur de laquelle circule un fluide non newtonien en régime laminaire. On considère aussi l'effet de la dissipation visqueuse. On présente une alternative pour déduire une extension de la solution de Lévêque jusqu'à des termes incluant le nombre de Brinkman. Les résultats obtenus sont comparés avec d'autres antérieurs pour montrer que notre expression finale, valable pour les tubes et les conduites plates, donnent des résultats très bons dans le domaine spécifié de validité des variables.

ERWEITERUNG DER LEVEQUE-LÖSUNG FÜR DIE WÄRMEÜBERTRAGUNG AN NICHT-NEWTONISCHE FLUIDE IN ROHREN UND FLACHEN KANÄLEN

Zusammenfassung—In der vorliegenden Arbeit wird das Problem der Berechnung der Wärmeübertragung durch die Wand eines Kanals behandelt, in dem ein nicht-newtonisches Fluid laminar strömt. Daneben wird auch der Einfluß der Wärmeerzeugung durch die viskose Dissipation betrachtet. In dieser Arbeit wird ein anderer Lösungsweg unter Berücksichtigung der Brinkman-Zahl eingeschlagen, um eine Erweiterung der Leveque-Lösung zu bekommen. Die gewonnenen Ergebnisse werden mit vorhandenen Ergebnissen verglichen, und es zeigt sich, daß unsere neue Gleichung, gültig für Rohre und flache Kanäle, ausgezeichnete Ergebnisse innerhalb des Gültigkeitsbereichs der angegebenen Variablen liefert.

УТОЧНЕННОЕ РЕШЕНИЕ ЛЕВЕКА ДЛЯ ТЕПЛОПЕРЕНОСА К НЕНЬЮТОНОВСКИМ ЖИДКОСТЯМ В ТРУБАХ И ПЛОСКИХ ТРУБОПРОВОДАХ

Аннотация—Дан анализ задачи об интенсивности теплопереноса через стенку канала при ламинарном течении неньютоновской жидкости с учетом тепловыделений из-за вязкой диссипации. Дано обобщенное решение типа Левека, учитывающее влияние числа Бринкмана. Полученные результаты сравниваются с известными. Показано, что выведенные формулы справедливы как для труб, так и для плоских каналов в диапазонах указанных выше параметров.